A critical damage criterion for creeping solids

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Creep damage is the progressive reduction in the material's ability to resist stress and is manifested as an increase in creep rate during tertiary creep eventually resulting in failure. Monkman–Grant relationship [1] that has contributed significantly to the development of creep fracture models [2], relates minimum creep rate $\dot{\varepsilon}_m$ and rupture life t_r as $\dot{\varepsilon}_m t_r = \text{constant} = C_{MG}$, for which, we provided a physical basis in terms of first order kinetics for creep [3–6]. In the present study, Monkman–Grant constant C_{MG} is henceforth referred to as 'Monkman–Grant Ductility (MGD)' that is the contribution of secondary creep strain to strain to failure, ε_f (Fig. 1). Monkman–Grant relation does not describe evolution of damage and its coupling to the deformation rate. The two approaches that describe this coupling are 'Continuum Creep Damage Mechanics (CDM)' by Kachanov–Rabotnov [7, 8] and 'Materials Properties Council (MPC)-Omega' by Prager [9]. CDM treats damage as an internal state variable and an important outcome is the creep damage tolerance factor λ [10, 11] defined as ratio of ε_f to MGD (Fig. 1) i.e., $\lambda = \varepsilon_f/(\dot{\varepsilon}_m t_r)$ and is a constant. λ is suggested [10] to be a better measure of creep ductility as it assesses the susceptibility of a material to localized cracking at strain concentrations [12], and thus can be considered as a material performance characteristic. Ashby and Dyson [11] provided a physical basis to CDM approach and demonstrated that each damage micromechanism, when acting alone, results in a characteristic value of λ. According to MPC-Omega approach [9], creep rate $\dot{\varepsilon}$ from its initial value $\dot{\varepsilon}_0$ increases with strain ε as $\dot{\varepsilon} = \dot{\varepsilon}_0 \exp(\Omega_p \varepsilon)$, where Ω_p is reciprocal of MGD, i.e., $\Omega_p = 1/(\dot{\varepsilon}_m t_r)$ for conditions showing negligible primary creep. The total damage coefficient Ω_p is the rate at which material's ability to resist stress is degraded by strain and is a material performance characteristic that is related to creep damage tolerance, i.e., higher the Ω_p, lesser is the resistance to creep damage.

In this paper, we identify that MGD is the critical strain at which true tertiary creep damage sets in and further introduces the concept of t_{MGD} (Fig. 1) as the time at which MGD is reached along the creep curve and the total secondary creep ductility is exhausted. t_{MGD} can be obtained as the time at which $\varepsilon - \varepsilon_p = \text{MGD}$, ε_p is the limiting primary creep strain, and for negligible ε_p as shown in Fig. 1, t_{MGD} is the time to reach MGD. Damage evolves along the creep curve and for any damage mechanism, we put forward that t_{MGD} is the time at which damage attains a critical level, beyond which its accelerated growth leads to failure. For a typical case of cavitation mechanism, it is shown that time to reach critical cavity size t_{CCS} matches well with t_{MGD}. We propose a *critical damage criterion* based on CDM as well as MPC-Omega method in terms of a unique relationship between t_{MGD} and t_r that depends only on λ. Further, we demonstrate its universal applicability for a wide range of materials and discuss its implications to engineering creep design.

In order to validate our proposition that t_{MGD} is the time at which damage attains a critical level, we analyse the mechanistic data reported by Davis and Williams [13] on α-iron at various temperatures (815–978 K) and stress levels (17.24–68.95 MPa). They observed a two stage tertiary creep behaviour; in the first stage, tertiary creep strain-time followed a $t^{4/3}$ law, whereas the second stage obeyed an exponential law. They concluded that the end of first stage corresponds to the time at which creep cavities attain a critical size and the second stage is the 'true tertiary creep'. Accordingly, the time corresponding to the end of first stage is designated as time to attain critical cavity size t_{CCS}. Creep curves that include secondary and tertiary creep regimes were generated using creep data as well as the tertiary creep strain–time relations given by Davis and Williams [13]. ε_f was obtained as the sum of ε_p, MGD and ε_t (cf. Fig. 1), where ε_t was calculated as $\varepsilon_t = A \exp[\beta(t_r - t_{ot})]$ and the constant reported value [14] of ε_p was taken as 0.051. From the generated creep curves, λ (discussed later) and t_{MGD} were determined; t_{MGD} was obtained as the time at which $\varepsilon - \varepsilon_p = \text{MGD}$. The plot of t_{CCS} vs. t_{MGD} in Fig. 2 demonstrates that t_{CCS} matches well with t_{MGD} and validates our proposition of t_{MGD} as the time at which creep damage attains a critical level. We like to extend such a validation for other damage mechanisms, but it has not been possible due to the lack of mechanistic data. Further, when two coupled mechanisms operate as in the case of nickel base superalloys [12], we suggest that t_{MGD} might correspond to the time at which cavitation intervenes the strain softening mechanism causing true tertiary creep damage.

Based on CDM approach, recently we proposed [15] a new relationship between t_{MGD} and t_r in terms of λ, and a brief description is given here. According to CDM, the evolution of deformation and damage is expressed in terms of internal state damage variable ω and for uniaxial stressing [12, 16] $\dot{\varepsilon} = \dot{\varepsilon}_0\ (\sigma/\sigma_0)^n$

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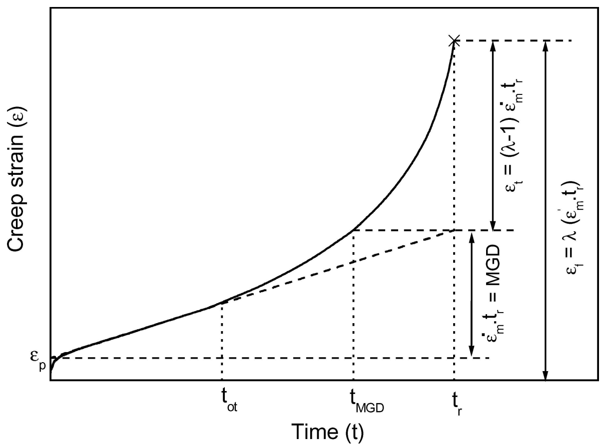

Figure 1 Schematic creep curve with negligible primary creep strain ε_p illustrating time to reach Monkman-Grant ductility t_{MGD}, time to onset of tertiary creep t_{ot}, damage tolerance factor λ and limiting tertiary creep strain ε_t.

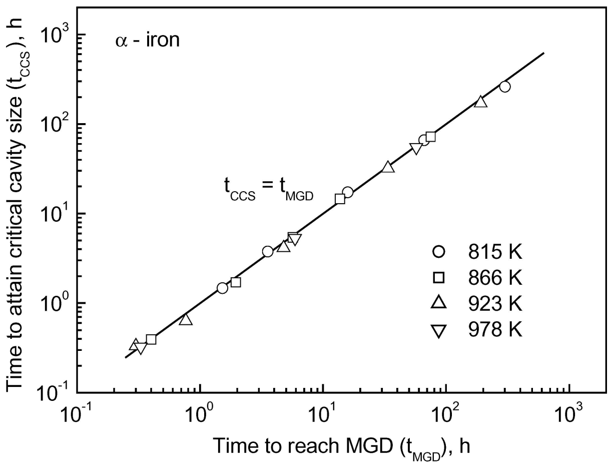

Figure 2 Demonstrates $t_{CCS} \approx t_{MGD}$ for creep cavitation micromechanism in α-Fe. Solid line is according to $t_{CCS} = t_{MGD}$ and symbols correspond to the experimental data.

$[1/(1-\omega)]^n$ and $\dot{\omega} = \dot{\omega}_0\,(\sigma/\sigma_0)^m\,[1/(1-\omega)]^m$, where $\dot{\varepsilon}_0$ and $\dot{\omega}_0$ are temperature dependent rate constants at a stress σ_0; n and m are constants. It is assumed that $\omega = 0$ when the material is in its undamaged state and $\omega = 1$ at rupture. Integration of these coupled equations at constant stress, for $m + 1 > n$ gives relation between strain fraction $\varepsilon/\varepsilon_f$ and time fraction t/t_r as $\varepsilon/\varepsilon_f = 1 - (1 - t/t_r)^{1/\lambda}$, where $\lambda = (m+1)/(m-n+1)$. For negligible ε_p (Fig. 1), $\varepsilon_t = (\lambda - 1)\dot{\varepsilon}_m t_r$, since $\lambda = \varepsilon_f/(\dot{\varepsilon}_m t_r)$ and $\varepsilon_t = \varepsilon_f - (\dot{\varepsilon}_m t_r)$. At $t = t_{MGD}$, creep strain $\varepsilon = \text{MGD} = \dot{\varepsilon}_m t_r$, and on substituting this in the equation relating $\varepsilon/\varepsilon_f$ and t/t_r, and on rearrangement we get $\varepsilon_t/\varepsilon_f = (1 - t_{MGD}/t_r)^{1/\lambda}$. In this equation, substituting for ε_t and λ, we obtain the *critical damage criterion* in terms of a universal relationship between t_{MGD} and t_r as

$$\frac{t_{MGD}}{t_r} = 1 - \left(\frac{\lambda - 1}{\lambda}\right)^{\lambda} = \text{constant} = f_{CDM}, \quad (1)$$

where f_{CDM} can be determined knowing the value of λ. Unlike t_{MGD} and t_r, t_{MGD}/t_r is independent of stress and temperature. We call the physically based Equation 1 as *critical damage criterion* because damage attains critical value at t_{MGD} when the criterion $t_{MGD} = f_{CDM}\,t_r$ is met, whereas t_{MGD} is the time at which creep damage attains a critical level. The theoretical plot of t_{MGD}/t_r vs. λ following Equation 1 is shown in Fig. 3, where t_{MGD}/t_r decreases with increasing λ and saturates at 0.63. This is in order since Equation 1 is of the functional form $y = 1 - [1 - (1/x)]^x$ and in the limit $x \to \infty$, $y = (1 - 1/e) = 0.63$.

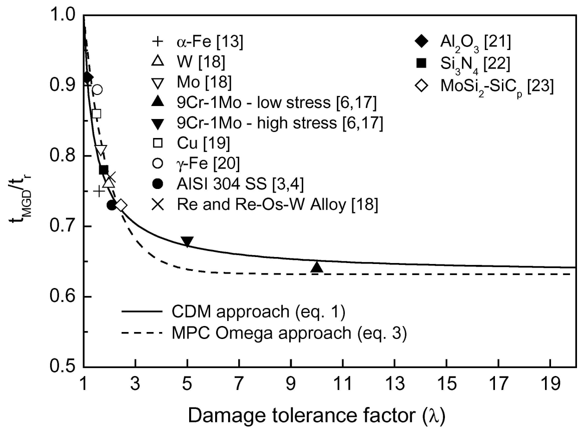

Figure 3 Validity of critical damage criterion (Equation 1) for various materials. Solid line is according to Equation 1 based on CDM approach, whereas broken line is according to Equation 3 based on MPC–Omega method. Symbols correspond to f_{EXP} values obtained from double logarithmic plot of t_{MGD} vs. t_r for different materials. Data for *W* is taken from Fig. 5.9, Mo from Fig. 8.13, Re and Re-Os-W alloy from Fig. 6.5 and Fig. 6.17B, respectively, from Ref. 18.

Along the lines of CDM approach, we also deduce the critical damage criterion based on MPC-Omega method [9]. For the sake of brevity, we start from the equation given by Prager (i.e., Equation 12 in Ref. [9]) for any time t and at t_r as

$$\frac{1}{\dot{\varepsilon}_m \Omega_P}\left(e^{-\Omega_P \varepsilon} - e^{-\Omega_P \varepsilon_f}\right) = (t_r - t), \quad (2)$$

where $\Omega_p = 1/\dot{\varepsilon}_m t_r$ and from the definition of λ, $\varepsilon_f = \lambda/\Omega_p$. Substituting for Ω_p and ε_f as well as the condition $\varepsilon = \text{MGD} = \dot{\varepsilon}_m t_r$ at $t = t_{MGD}$ in Equation 2 and on rearrangement, the relationship between t_{MGD} and t_r in terms of λ can be deduced as

$$\frac{t_{MGD}}{t_r} = 1 - e^{-1} + e^{-\lambda} = \text{constant} = f_{MPC}. \quad (3)$$

In Equation 3, when $\lambda \gg 1$, $t_{MGD}/t_r = (1 - 1/e) = 0.63$ and is in accordance with Equation 1 for large values of λ. Using Equation 3, f_{MPC} can be calculated knowing the value of λ. The theoretical plot of Equation 3 is also shown in Fig. 3 illustrating that t_{MGD}/t_r decreases with increasing λ and saturates at 0.63. It is evident from Fig. 3 that both the Equations 1 and 3 are identical in implying the uniqueness of the relationship between t_{MGD} and t_r. Thus MPC-Omega method further strengthens the proposed damage criterion (i.e., Equation 1) and reinforces that t_{MGD}/t_r depends only on λ.

We demonstrate the validity of critical damage criterion for a wide range of materials. Analysis of results

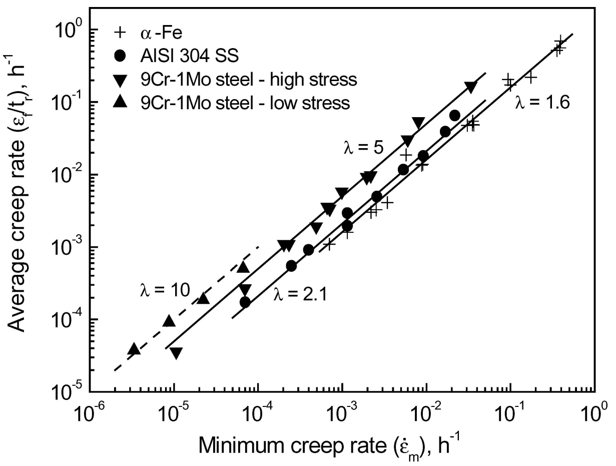

Figure 4 Illustrates the constancy of λ for α-Fe, 9Cr-1Mo steel and AISI 304 stainless steel. λ is determined as the intercept at $\dot{\varepsilon}_m = 1$ in the double logarithmic plots of ε_f/t_r vs. $\dot{\varepsilon}_m$, since $\varepsilon_f/t_r = \lambda\dot{\varepsilon}_m$.

on 9Cr-1Mo ferritic steel [6, 17] and AISI 304 stainless steel [3, 4] showing the constancy of λ is presented in Fig. 4. Unlike 304 stainless steel ($\lambda = 2.1$), 9Cr-1Mo steel exhibited separate constant values of $\lambda = 10$ and 5 for low and high stress regimes, respectively. Published tensile creep data [13, 18–23] for various materials such as pure metals, ceramics and composite of intermetallic silicide were also analysed for λ, t_{MGD} and t_r. λ was found to be constant for a given material and this for a typical case of α-Fe [13] is shown in Fig. 4. For different materials, logarithmic plots of t_{MGD} vs. t_r obeyed $t_{MGD} \propto t_r$ (i.e., $t_{MGD} = f_{EXP}\, t_r$) and the observed f_{EXP} values are shown as symbols in Fig. 3. The plot of t_{MGD} vs. t_r in Fig. 5 demonstrates the validity of damage criterion (Equation 1) for various materials and f_{CDM} values ranged from 0.65 for 9Cr-1Mo steel to 0.91 for Al_2O_3. Unlike Fig. 3, the difference in f_{CDM} values is not seen in Fig. 5 due to logarithmic representation of the plot. We emphasise that the damage mechanism specific nature of damage criterion comes from f_{CDM} which is related to λ, as any change in damage mechanism changes the value of λ [11].

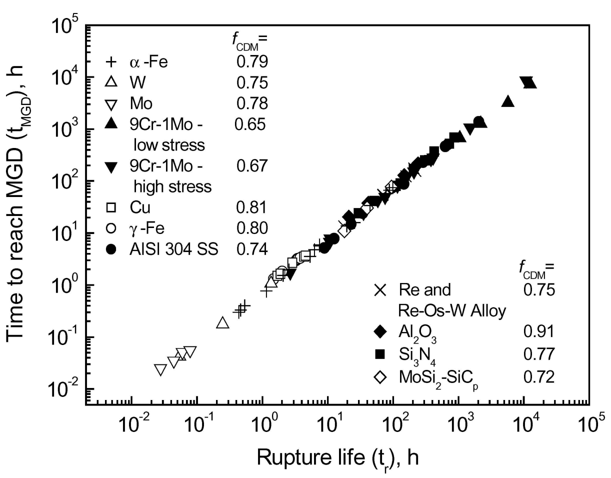

Figure 5 Variation of t_{MGD} with t_r demonstrating the validity of critical damage criterion (Equation 1) with f_{CDM} values calculated using respective values of λ for different materials. Symbols represent experimental data obeying Equation 1.

The damage criterion has important implications. First is that t_{MGD} conceptually divides creep curve into two parts (Fig. 1), and this is something similar to plastic instability in tension dividing the stress-strain curve into uniform and non-uniform deformation regimes. The second is its importance to engineering creep design. The ratio t_{MGD}/t_r saturating to ∼2/3 (Fig. 3) provides a physical basis for the factor of safety of 67% employed on stress to cause rupture in 10^5 hr to arrive at the design allowable stress [24, 25]. Further, as critical damage sets in and minimum ductility is assured up to t_{MGD}, we suggest that the stress to cause t_{MGD} in 10^5 hr can be considered as a new design criterion. The proposed damage criterion also has its implication to Robinson's life fraction damage rule [25, 26], which states that under non steady stress and temperature conditions, failure occurs when $\Sigma\Delta t_i/t_{ri} = 1$, where Δt_i is the time spent at any given stress and temperature and t_{ri} is the rupture life under those conditions. When $\Sigma\Delta t_i/t_{ri}$ either equals or exceeds unity, it can be conveniently used as a conservative basis for life prediction. For situations when it predicts non conservative values of remnant life when $\Sigma\Delta t_i/t_{ri} < 1$, we propose a modified life fraction rule as $\Sigma\Delta t_i/t_{MGDi} = 1$, which is conservative. It can be shown that $\Sigma\Delta t_i/t_{MGDi} = 1$ leads to $\Sigma\Delta t_i/t_{ri} < 1$ as $t_{MGD} = f_{CDM} t_r$ (i.e., $\Sigma\Delta t_i/t_{MGDi} = (1/f_{CDM})\ \Sigma\Delta t_i/t_{ri} = 1$ and $\Sigma\Delta t_i/t_{ri} = f_{CDM} < 1$), since it is reasonable to assume that the f_{CDM} remains constant (i.e., λ is constant) in a given stress-temperature domain.

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